AD-A195 291 MOMENT EXERTED ON A CONING PROJECTILE BY A SPINNING LIQUID IN A CYLINDRIC..(U) ARMY BALLISTIC RESEARCH LAB ABERDEEN PROVING GROUND MD G R COOPER JUN 88 BRL-MR-3677 1/1

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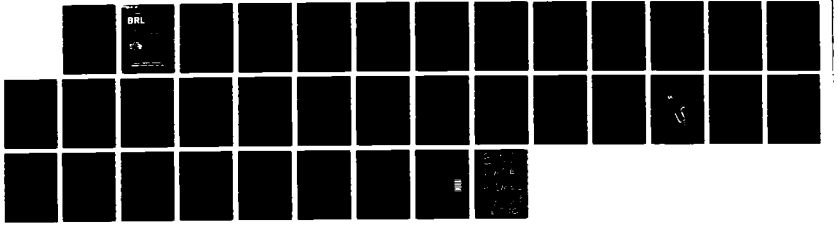

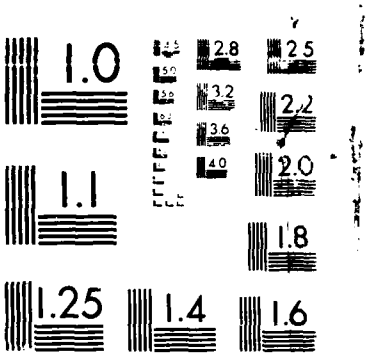

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MEMORANDUM REPORT BRL-MR-3677

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MOMENT EXERTED ON A CONING PROJECTILE BY A SPINNING LIQUID IN A CYLINDRICAL CAVITY CONTAINING A POROUS MEDIUM

GENE R. COOPER

JUNE 1988

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY

ABERDEEN PROVING GROUND, MARYLAND

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) BRL-MR-3677	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION U.S. Army Ballistic Research Laboratory 6b. OFFICE SYMBOL (If applicable) SLCBR-LF	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Aberdeen Proving Ground, MD 21005-5066	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U.S. Army Ballistic Research Laboratory 8b. OFFICE SYMBOL (If applicable) SLCBR-DD-T	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) Aberdeen Proving Ground, MD 21005-5066	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. 62618A PROJECT NO. 1L1 62618AH80 TASK NO. WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Moment Exerted on a Coning Projectile by a Spinning Liquid in a Cylindrical Cavity Containing a Porous Medium (U)

12. PERSONAL AUTHOR(S)
Cooper, Gene R.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Memorandum Report	FROM _____ TO _____	1988 April	29

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Cylindrical Cavity, Porous Media, Inviscid, Rotating Liquid
01	01		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

White phosphorous (WP) impregnated felt wedges are used as a payload in the M825 improved smoke projectile. An assumption made in this work is that the WP is in a liquid state (i.e., temperature > 44 degrees C) where such payloads have been seen to cause flight instabilities. The analytical results given here formulate an initial effort to gain an understanding of the dynamics of a projectile interacting with a WP/felt payload. The analytical methods used here are a simple extension of previous methods used to describe bulk-filled liquid payloads. Moments are predicted due to an inviscid liquid moving through a ridged porous medium which is confined to a spinning cylindrical cavity undergoing coning motion. A drag term is added to the classical Stewartson theory which is used to describe the flow in the porous media. The cylindrical cavity is assumed to consist of several chambers of circular cross section and uniform height, each separated by solid endcaps. This porous media theory is used to calculate the total liquid side moments exerted by all the chambers in the cylinder. Results are presented for a range of coning frequencies, fineness ratios, and porous drag coefficients.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Gene R. Cooper	22b. TELEPHONE (Include Area Code) (301)-278-2057 22c. OFFICE SYMBOL SLCBR-LF-A

DD Form 1473, JUN 86 *Previous editions are obsolete.*

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

ACKNOWLEDGMENTS

The author is deeply indebted to Dr. William P. D'Amico for suggesting this study. Dr. D'Amico spent many hours discussing the topics of this report with the author, as well as, editing the manuscript. The author also wishes to acknowledge the many helpful comments and suggestions received from Dr. Charles H. Murphy.

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I. INTRODUCTION

Predicting the moment exerted by a liquid payload in a spinning and coning projectile is a problem of considerable interest to the Army. Stewartson[1] considered the linear problem of calculating the payload moment through the use of separation of variables and eigenvalue expansions for an inviscid liquid in a cylindrical cavity. First order viscous boundary layer corrections to the Stewartson theory were carried out by Wedemeyer[2] and Murphy.[3] A method for calculating the linear liquid moment using the full viscous equations with boundary layer corrections confined only to the endcaps was also presented by Gerber and Sedney.[4] They have recently extended this theory to eliminate the boundary layer correction at the endcaps.[5]

Liquid payloads contained in a highly permeable material have also been of interest to the Army for some time. Laboratory tests and flight tests have shown that a highly permeable medium can significantly reduce the spin-up time of a liquid payload.[6] Flight stability for liquid saturated permeable payloads has also been examined by D'Amico.[7]

This report extends the Stewartson problem by considering a cylindrical cavity filled with a permeable medium that is impregnated with an inviscid liquid. A further modification is introduced by segmenting the cavity, along the symmetry axis, into a sequence of equal length cylinders. Each of these cylinders is separated by impermeable endcaps. The porous media is modeled by a drag term, which is proportional to the velocity field, added to the linearized Euler equations. This analysis examines the induced liquid moment as a function of parameters found by Stewartson plus parameters describing the porous media and the number of segments in the cylindrical cavity.

The nomenclature in this report is the same as that used by Murphy.[3] In particular, this means that the liquid moment can be represented as the complex quantity:

$$\text{Transverse Moment} = m_L a^2 \dot{\phi}^2 \tau \left[C_{LSM} + i\, C_{LIM}\right] K_c\, e^{i\phi_c} \qquad (1.1)$$

where

m_L is the mass of liquid in a fully-filled cavity,

a is the maximum radius of the container,

$\dot{\phi}$ is the inertial spin rate of the container along the symmetry axes,

τ is the ratio of coning rate to spin $\dot{\phi}_c/\dot{\phi}$,

C_{LSM} is the liquid side moment coefficient,

C_{LIM} is the liquid in-plane moment coefficient,

K_c is sin α_c is the precession angle.

II. LIQUID MOMENT

Two coordinate systems, each with X-axis along the projectile symmetry axis, are used: the missile-fixed (X, Y, Z) system and the non-rolling $X\tilde{Y}\tilde{Z}$ system with the $\tilde{Z}$-axis initially pointing downward. Introduce an earth-fixed axes (X_e, Y_e, Z_e) with X_e-axis in the direction of the velocity vector and Z_e downward. Let a unit vector in the direction of the X-axis have earth-fixed components (n_{XE}, n_{YE}, n_{ZE}). The angle of attack $\tilde{\alpha}$ in the (X, $\tilde{Y}$, $\tilde{Z}$) system is the projection on the $X\tilde{Z}$-plane of the angle between the X-axis and the velocity vector. The angle of sideslip $\tilde{\beta}$ is the projection of the same angle onto the $X\tilde{Y}$-plane.

The kinematic behavior of the spinning projectile is the sum of two coning motions:[3]

$$\tilde{\xi} = \tilde{\beta} + i\,\tilde{\alpha} = K_1\, e^{i\phi_1} + K_2\, e^{i\phi_2} \tag{2.1}$$

where

$$\ln (K_j/K_{j0}) = \varepsilon_j\, \tau_j \phi$$

$$\phi_j = \phi_{j0} + \tau_j\, \phi$$

$$\phi = \dot{\phi} t$$

($\dot{\phi} \geq 0$) is the spin rate and K_{j0}, ϕ_{j0} are constants.

III. EQUATIONS OF LIQUID MOTION IN POROUS MEDIA

The following analysis models the steady state response of a liquid flowing in a porous medium confined to a cylindrical cavity contained in a spinning and coning projectile. The objective of this theory is to predict the liquid moment resulting from coning or spiral motion which is specified by:[3]

$$\tilde{\xi} = \hat{K}\, e^{s\phi} \tag{3.1}$$

where

$$\hat{K} = K_{j0}\, e^{i\phi_{j0}}, \qquad j = 1 \text{ or } 2$$

$$s = (\varepsilon_j + i)\tau_j$$

The conservation equations governing the motion of the confined liquid are the continuity equation and a modified Euler-momentum equation. The modification is one that is commonly used to describe the flow of liquid through a porous medium. This consists of an additional term given by:

$$\vec{D}_r = -\frac{\mu}{\kappa}(\vec{V}_R) = -\rho_L a\dot{\phi}^2 C_r\left(\frac{\vec{V}_R}{a\dot{\phi}}\right) \tag{3.2}$$

where

μ is the dynamic viscosity

κ is the porosity (dimensions of length^2)

$\vec{V}_R$ is the velocity of liquid relative to the porous medium

ρ_L is liquid density

$\vec{D}_r$ is a pressure gradient induced by resistance of the porous media to fluid flow.[8] $C_r = \frac{\mu}{\rho_L K\dot{\phi}}$ is a dimensionless coefficient which is a measure of this pressure gradient.

Physical reasons for using such equations can be found from the arguments used in establishing Darcy's Law.[9]

Let (r,θ,x) be cylindrical polar coordinates fixed to the earth frame and (V,W,U) be the corresponding components of velocity. For small angles, the position vector of any point in the projectile has components[3]

$$r = \tilde{r} - K_j \tilde{x}\cos(\phi_j - \tilde{\theta}) \tag{3.3}$$

$$x = \tilde{x} + K_j \tilde{r}\cos(\phi_j - \tilde{\theta}) \tag{3.4}$$

$$\theta = \tilde{\theta} + O(K_j^2) \tag{3.5}$$

where tilda (~) quantities are measured in the non-rolling system. Equations (3.3 - 3.5) lead to the velocity components

$$V_x = R\{\dot{\phi}(s - i)r\hat{K}\, e^{s\phi - i\theta}\} \tag{3.6}$$

$$V_r = -R\{\dot{\phi}(s - i)x\hat{K}\, e^{s\phi - i\theta}\} \tag{3.7}$$

$$V_\theta = \dot{\phi} r + R\{i\dot{\phi}(s - i) x \hat{K}\, e^{s\phi - i\theta}\} \tag{3.8}$$

where

$$R\{\} = [\ \{\} + \overline{\{\}}\ ]/2 = \text{Real part of } \{\}\ .$$

Let the velocity, $\vec{V}$, and pressure, p, fields have the form:

$$\vec{V} = \vec{q} + \dot{\phi}\, \hat{e}_x \times \vec{r}$$

$$P = p + \frac{\rho_L \dot{\phi}^2 r^2}{2} \tag{3.9}$$

where $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_x$ are the unit vectors in the (r,θ,x) directions. The variables $\vec{q} = (v,w,u)$, and p are small perturbations of $O(K_j)$.

Non-dimensionalizing all lengths by the cylinder radius a and velocities by $a\dot{\phi}$, plus assuming periodic disturbances of the form

$$(v,w,u,p) = (v\ (r,x),\ w\ (r,x),\ p\ (r,x))\ e^{s\phi - i\theta} \tag{3.10}$$

allows the continuity and momentum equations to be written as

$$\frac{\partial v}{\partial r} + \frac{v}{r} - \frac{iw}{r} + r\,\frac{\partial u}{\partial x} = 0 \tag{3.11}$$

$$(s - i)\,\vec{q} + 2\,\hat{e}_x \times \vec{q} = -\nabla p - C_r[\vec{q} + (s - i)(x,-ix,-r)]. \tag{3.12}$$

The components of Eq. (3.12) take the form

$$\gamma v - 2w + \frac{\partial p}{\partial r} = -\,(s-i) \times C_r \tag{3.13}$$

$$\gamma w + 2v - \frac{ip}{r} = i\,(s-i) \times C_r \tag{3.14}$$

$$\gamma u + \frac{\partial p}{\partial x} = (s-i)\ r\ C_r \tag{3.15}$$

where $0 \leq r \leq 1$, $h + f \leq x \leq h - f$ f = fineness ratio

$$\gamma = s\text{-}i + C_r$$

and h is the center of mass location along the symmetry axis. The non-homogeneous terms of Equations (3.13 - 3.15) suggest using the transformations

$$v = v_H - \frac{(s-i) \times C_r}{(\gamma + 2i)} \tag{3.16}$$

$$w = w_H + \frac{i\ (s-i) \times C_r}{(\gamma + 2i)} \tag{3.17}$$

$$u = u_H + \frac{(s-i)\ r\ C_r}{\gamma} \tag{3.18}$$

in the equations of motion which reduces them to a single equation for the pressure p

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{p}{r^2} = -\frac{(\gamma^2+4)}{\gamma^2}\frac{\partial^2 p}{\partial x^2}\ . \tag{3.19}$$

Similarly, the solution p is related to the following physical quantities of interest:

$$v_H = \frac{1}{\gamma^2 + 4}\left[\frac{2ip}{r} - \gamma\frac{\partial p}{\partial r}\right] \tag{3.20}$$

$$w_H = \frac{1}{\gamma^2 + 4}\left[\frac{i\gamma p}{r} + 2\frac{\partial p}{\partial r}\right] \tag{3.21}$$

$$u_H = -\frac{1}{\gamma}\frac{\partial p}{\partial x}\ . \tag{3.22}$$

The cylindrical cavity is assumed to consist of N chambers with circular cross section and height $\Delta = 2f/N$, each separated by impenetrable endcaps as depicted in Figure 1.

The boundary conditions for Eq. (3.19) are the normal velocity of the fluid at the boundary must equal the normal velocity of the wall and all flow variables remain finite. This means:

$$\frac{\partial \hat{p}}{\partial x} = 0 \qquad \text{at } x = h - f + n\left(\frac{2f}{N}\right) \qquad n = 0,1,\ldots N \tag{3.23}$$

$$2i\hat{p} - \gamma \frac{\partial \hat{p}}{\partial r} = -2xs\,(s-i)\,(s-3i+C_r) \qquad \text{at } r = 1 \tag{3.24}$$

$$\hat{p} = 0 \quad \text{at } r = 0 \tag{3.25}$$

where $$\hat{p} = p + xr\,(s-i)^2. \tag{3.26}$$

This boundary value problem is similar to the problem considered by Stewartson[1] and can be shown to have the solution

$$p = -xr\,(s-i)^2 + xs\,(s-i)\,[2(h-f) + (2n+1)\,\Delta]\,r + \frac{16fs\,(s-i)(\gamma-2i)}{\pi^2 N} \sum_{k=\text{odd}}^{\infty} \frac{J_1(\lambda_k r)\cos\frac{\pi k}{\Delta}(x-h+f-n\Delta)}{k^2\,[(2i+\gamma)\,J_1(\lambda_k) - \gamma\,\lambda_k\,J_0(\lambda_k)]} \tag{3.27}$$

where $$\lambda_k^2 = -\frac{(\gamma^2+4)\,N^2\,k^2\,\pi^2}{\gamma^2\,4\,f^2}; \qquad N = 1,2,3,\ldots \quad n = 0,1,\ldots N$$

and the J_n are the Bessel functions of the first kind. If γ is pure imaginary, the transcendental denominator in Eq. (3.27) will have zeroes. This resonant condition occurs whenever $\tau\,\varepsilon + C_r = 0$ and τ is one of the inviscid Stewartson[1] eigenfrequencies.

IV. LIQUID MOMENT EQUATIONS

The moment induced by the liquid contained in the segmented cavity is calculated from the time derivative of the angular momentum field. Non-dimensionalizing this moment with $2\pi a^3 f \rho_L$ allows the moment in the YX plane to be expressed as a single complex quantity[3]

$$M_y + i\, M_z = \tau\, C_{LM}\, \hat{K}\, e^{s\phi} \tag{4.1}$$

where $C_{LM} = C_{LSM}\,(\tau,\varepsilon,C_r,f) + i\, C_{LIM}\,(\tau,\varepsilon,C_r,f)$.

The unit vectors for the earth-fixed cylindrical coordinates, $(\hat{e}_r, \hat{e}_\theta, \hat{e}_x)$, can be written in the same complex notations in terms of $(\hat{e}_y, \hat{e}_z, \hat{e}_x)$

$$\hat{e}_r = \hat{e}_y \cos\theta + \hat{e}_z \sin\theta \;<=>\; e^{i\theta} \tag{4.2}$$

$$\hat{e}_\theta = -\hat{e}_y \sin\theta + \hat{e}_z \cos\theta \;<=>\; ie^{i\theta}. \tag{4.3}$$

Substituting these into the moment integral and using the Reynolds Transport Theorem[10] gives the following expression for the liquid moment coefficient:

$$\tilde{\tau}\, C_{LM} = \frac{1}{2f} \sum_{n=0}^{N1-} \int_{h-f+n\Delta}^{h-f+(n+1)\Delta} \int_0^1 \{(\tilde{x}\hat{e}_x + \tilde{r}\hat{e}_r) \times [(s-i)\vec{\tilde{q}} + 2\hat{e}_x \times \vec{\tilde{q}}] - i[\tilde{r}^2 - 2\tilde{x}^2]\}\, \tilde{r}\, d\tilde{r}\, d\tilde{x}. \tag{4.4}$$

Using Eqs. (3.16-3.18 and 3.20-3.22) with the aid of Eq. (3.27) permits writing the last equation as

$$\tilde{\tau}\, C_{LM} = \frac{i}{12} \left[\frac{4[1-C_r(s-i)]\,[(s^2+1)\,(3h^2+f^2)]C_r}{(\gamma+2i)} + \frac{3[1+C_r(s-i)](s-i)^2 C_r}{\gamma} - 12h^2 - 4f^2 + 3 \right] + \frac{2is}{(\gamma+2i)} \left[\frac{(s^2+1)\,(3h^2+f^2)}{3} - 2\frac{s(s-i)f^2}{N^2} \right] - \frac{128\, i\, (s-i)\,(\gamma-2i)s^2f^2}{(\gamma+2i)\, N^4\, \pi^4} \sum_{k=odd}^{\infty} \frac{J_1(\lambda_k)}{k^4[(\gamma+2i)\, J_1(\lambda_k) - \gamma\lambda_k\, J_0(\lambda_k)]}; \tag{4.5}$$

$N = 1,2,3,\ldots$

Murphy[3] derived the frozen liquid values of C_{LSM} and C_{LIM} and these are given by

$$C_{LSM} = \frac{\varepsilon}{2}\left[1 - \tau\left[1 + \frac{4(3h^2 + f^2)}{3}\right]\right] \tag{4.6}$$

$$C_{LIM} = \frac{1}{2} + \frac{\tau(\varepsilon^2 - 1)}{12}\left[3 + 12h^2 + 4f^2\right] . \tag{4.7}$$

For very slow motion ($\tau \to 0$) the liquid moment should approach these values. Equation (4.5) shows that the limiting value of C_{LIM} as $\tau \to 0$ equals $(\varepsilon + i)/2$. This agrees with the forzen liquid results when $\tau = 0$. When $C_r \to \infty$ the liquid should act like a frozen liquid for all values of τ. It is easy to see that the liquid moment coefficients given by Equation (4.5) approach (in the limit of $C_r \to \infty$ for λ_k not a resonance value) the values for a frozen liquid. The same limits, Equations (4.6 - 4.7), are also found when C_r and τ are arbitrary but N becomes infinitely large.

V. SOLUTION METHOD

The equations of sections III and IV need to be solved over a wide range of τ for specific values of D_ρ, f, and N. This requires calculating the ratio $J_0(\lambda_k)/J_1(\lambda_k)$ of the zero and first order Bessel functions. For values of k such that $|\lambda_k| \leq 50$, the above ratio is obtained by simply dividing the values found from power series expansions of each Bessel function. For larger values of $|\lambda_k|$, an asymptotic expansion of the Bessel function ratio was used.[11] Equation (4.5) is then used to find values of C_{LSM} and C_{LIM}. Experience has shown that $k \leq 20$ is sufficient to produce converged solutions. All computations were carried out on a VAX-8600 computer.

VI. DISCUSSION

Figures 2 and 3 present plots of C_{LSM} and C_{LIM} as functions of frequency for f = 1.5, C_r = 3.0, N = 1, and ε = 0 and 0.02. For zero damping, the maximum side moment is slightly larger than the maximum side moment for a small amount of undamping given by ε = 0.02. The in-plane moment remains relatively unchanged due to the presence of the same undamping. This indicates that C_{LM} is insensitive to ε for ε near zero.

It was mentioned earlier that C_{LSM} and C_{LIM} approach the limiting values for a frozen liquid, Eqs. (4.6-4.7), as $C_r \to 0$. An example of this for f = 2, N = 1 and ε = 0 is exhibited in Figures 4 and 5. Similar results are shown in Figures 6 and 7 for an increasing number of chambers N and a fixed C_r = 3.

Generally, it has been found that the frozen liquid limit is approached quite rapidly with increasing values of N for typical values of C_r.

If τ_o is a Stewartson eigenfrequency for the particular values f_o, k_o, and N_o, then a resonant condition will occur at $\tau = \tau_o$ whenever

$$\frac{f}{k\,N} = \frac{f_o}{k_o\,N_o} \qquad \begin{matrix} k_o,\ k = 1,3,5,\ldots \\ N_o,\ N = 1,2,3,\ldots \end{matrix} \tag{6.1}$$

For an example of this phenomenon, consider the case of $k_o = 1$, $f_o = 2$, $N_o = 1$, $C_r = 3$, and $\varepsilon = 0$. These parameter values make the ratio of Equation (6.1) equal to 2 and the first eigenfrequency $\tau_o = 0.5102$. Figure 8 shows C_{LSM} as a function of τ for $k = 1$, $f = 10$, and $N = 1,5,10$. The increase in C_{LSM} for $N = 5$ is due to the eigenfrequency $\tau_o = 0.5102$ since the ratio in Eq. (6.1) is again equal to 2. Hence for a fixed τ, an increase in N can cause C_{LSM} to increase. The eigenvalues for the $N = 1,10$ cases lie outside the range of τ given in Figure 8. C_{LSM} therefore decreases monotonically to the frozen liquid value, which is zero for $\varepsilon = 0$, when N becomes sufficiently large.

The 155mm M825 projectile is shown in Figure 9. This projectile carries a canister that is loaded with white phosphorous (WP) impregnated felt wedges. Four angular ribs produce longitudinal quadrants within which felt wedges (116 per canister) are loaded. Aluminum foil spacers are located between each wedge (as shown in Figure 9) and could produce a compartmentalization or segmentation as modeled in this theory. When the WP is liquid (temperatures above 44 deg C), flight instabilities have been recorded. Recent flight tests have shown that the flight stability of the projectile can be improved by using felt wedges with outer diameters that produce interference fits when loaded into the canister.[12] The model given in this report is directed at a fundamental understanding of the payloads used in the M825. The inclusion of a drag force on the liquid caused by the permeable media provides a first step in understanding the physics of these payloads.

Scheidegger[13] tabulated permeabilities of various substances and stated a range for hair felt as $8.3 \times 10^{-6}\ cm^2 < \kappa\ 1.2 \times 10^{-5}\ cm^2$. For an M825-type payload, nominal values for spin rate and kinematic viscosity are 100 Hz and 0.015 cm^2/sec. Hence, a median drag coefficient, C_r, is $2\pi/15$ or approximately 1/2. Experiments should be conducted to determine the permeability, κ, for the type of felt and packing used in the M825 projectile. Investigations must be conducted to verify if a linear, homogeneous formulation, using Darcy's Law, is appropriate for spinning liquids. The present analysis extends prior theories for liquid payloads and produces a simple model from which a clear physical understanding is obtained. Additions to the present effort could include the effects of fluid interaction with solid boundaries, radial variations in C_r and the nonisotropic character of C_r.

VII. CONCLUSIONS

The liquid moment coefficients are computed for cylindrical cavities which are fully-filled with a permeable medium and impregnated with an inviscid liquid. These coefficients reflect the results of segmenting a given cavity into N chambers with uniform height. The coefficients are then calculated as functions of coning frequency, fineness ratio, and the parameter C_r used to represent the physics of a liquid flowing through the permeable medium.

If the range of coning frequencies does not contain any eigenfrequencies, then the calculations plus theory indicate that the side moment approaches the value for a frozen liquid as $C_r \to 0$ and/or N becomes large. However, a significant increase in the side moment can occur for a particular N by choosing the fineness ratio such that an eigenfrequency moves into the frequency range of interest.

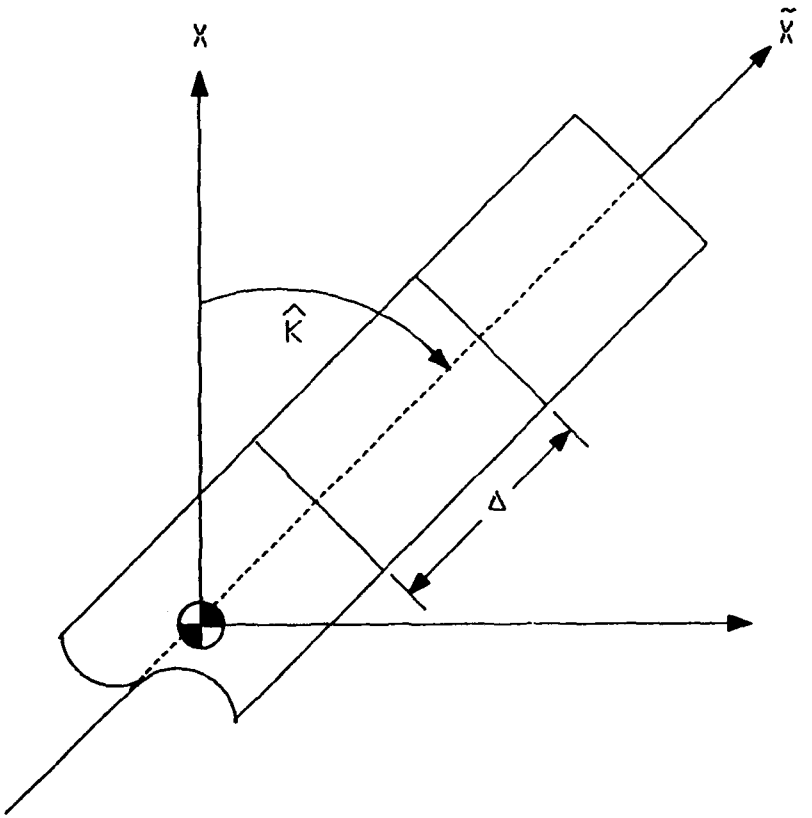

Figure 1. Diagram showing a typical cylindrical segment with impenetrable endcaps.

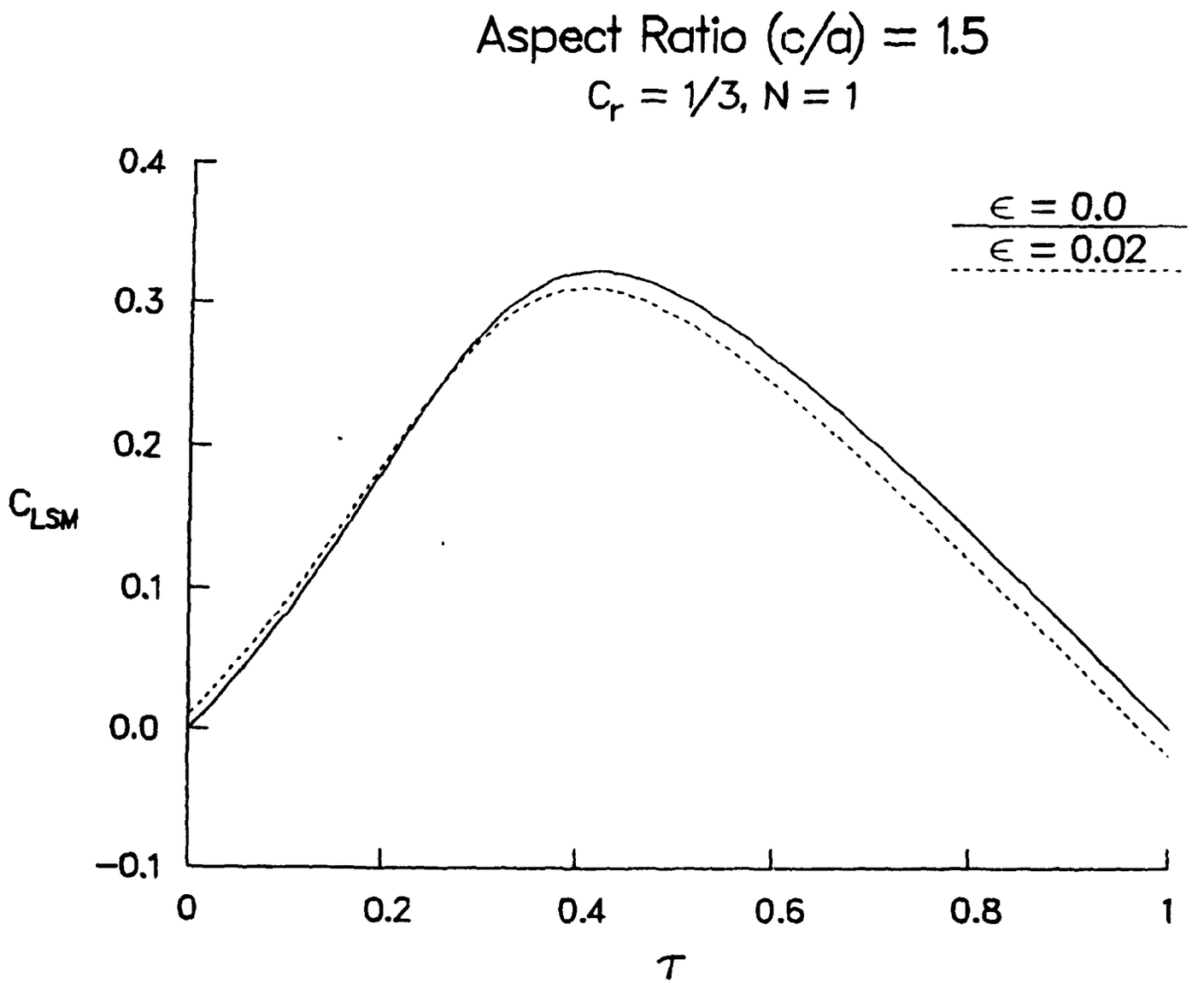

Figure 2. C_{LSM} versus τ for f = 1.5, C_r = 3, ϵ = 0.0, 0.02.

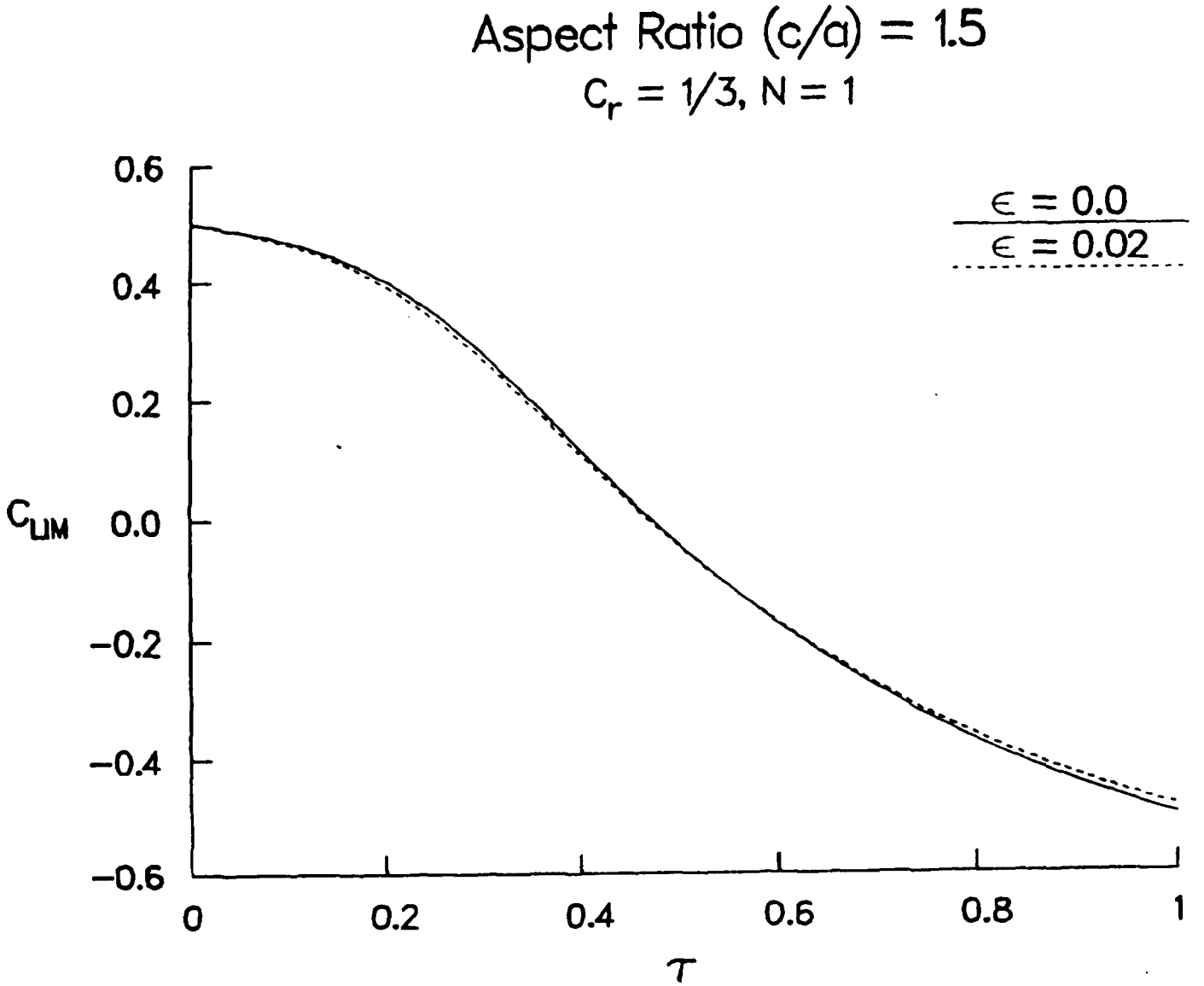

Figure 3. C_{LIM} versus τ for f = 1.5, C_r = 3, ϵ = 0.0, 0.02.

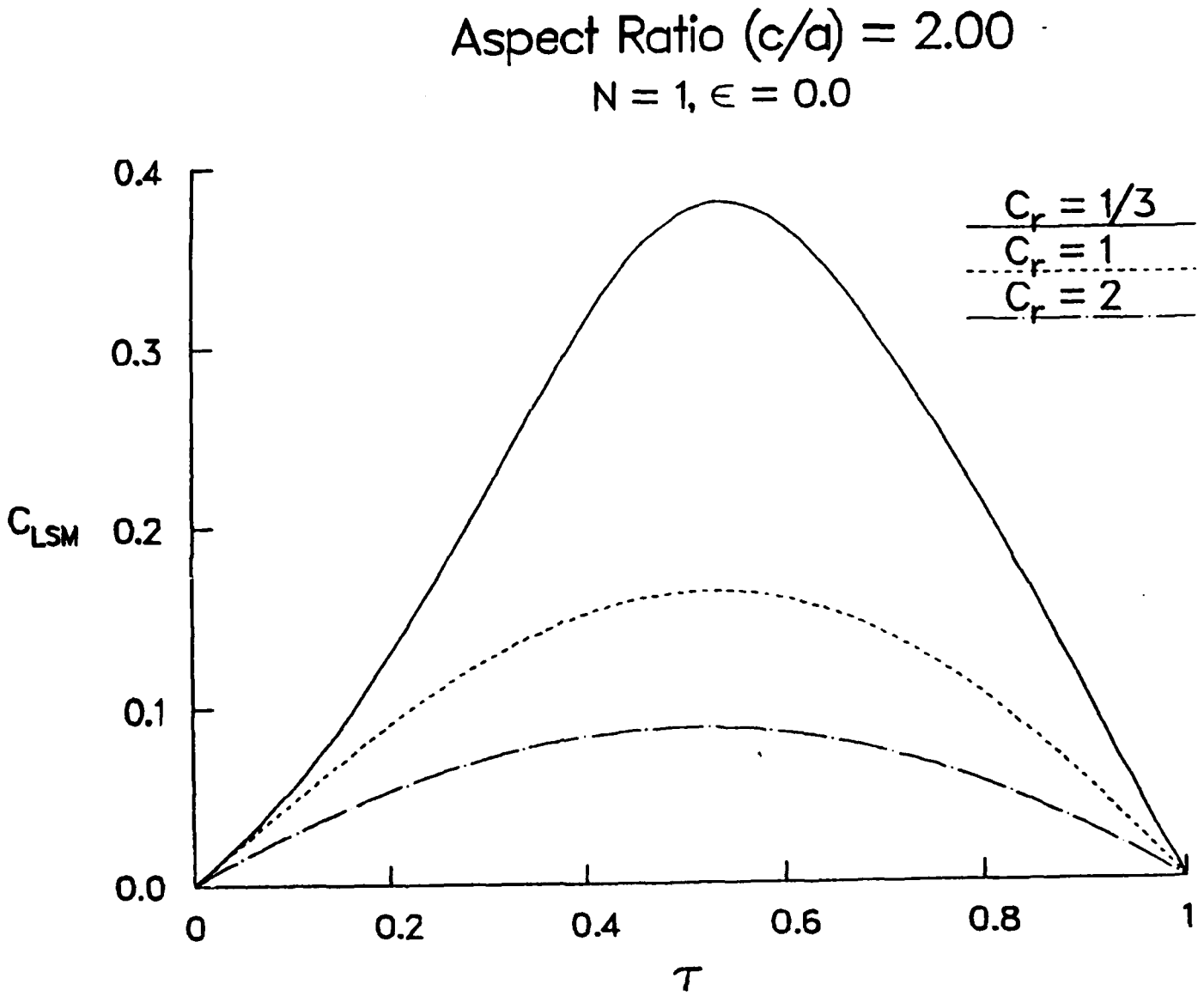

Figure 4. Comparison of C_{LSM} versus τ for f = 2, N = 1, ϵ = 0, C_r = 3, 1, 0.5.

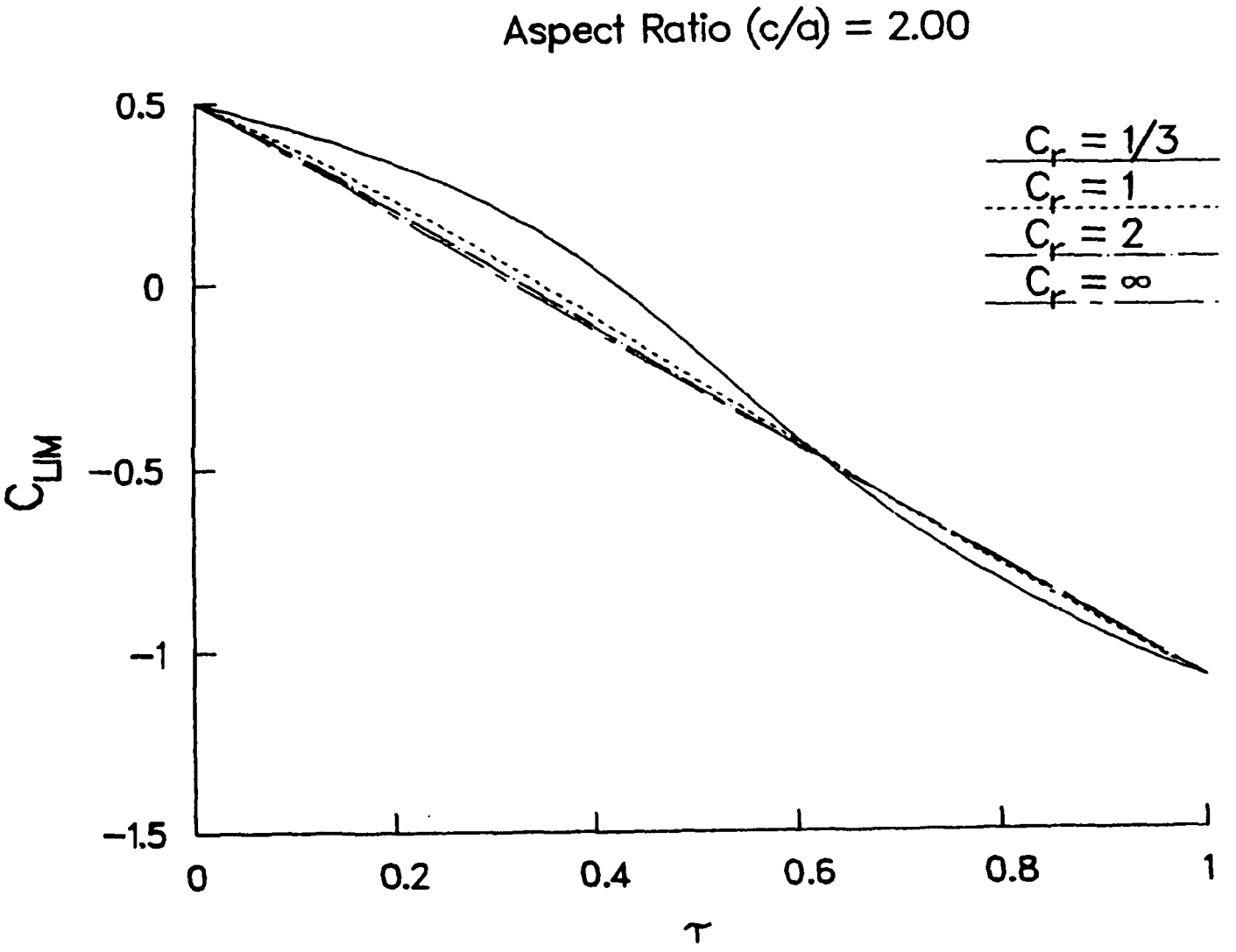

Figure 5. Comparison of C_{LIM} versus τ for f = 1, N = 1, ε = 0, C_r = 3, 1, 0.5.

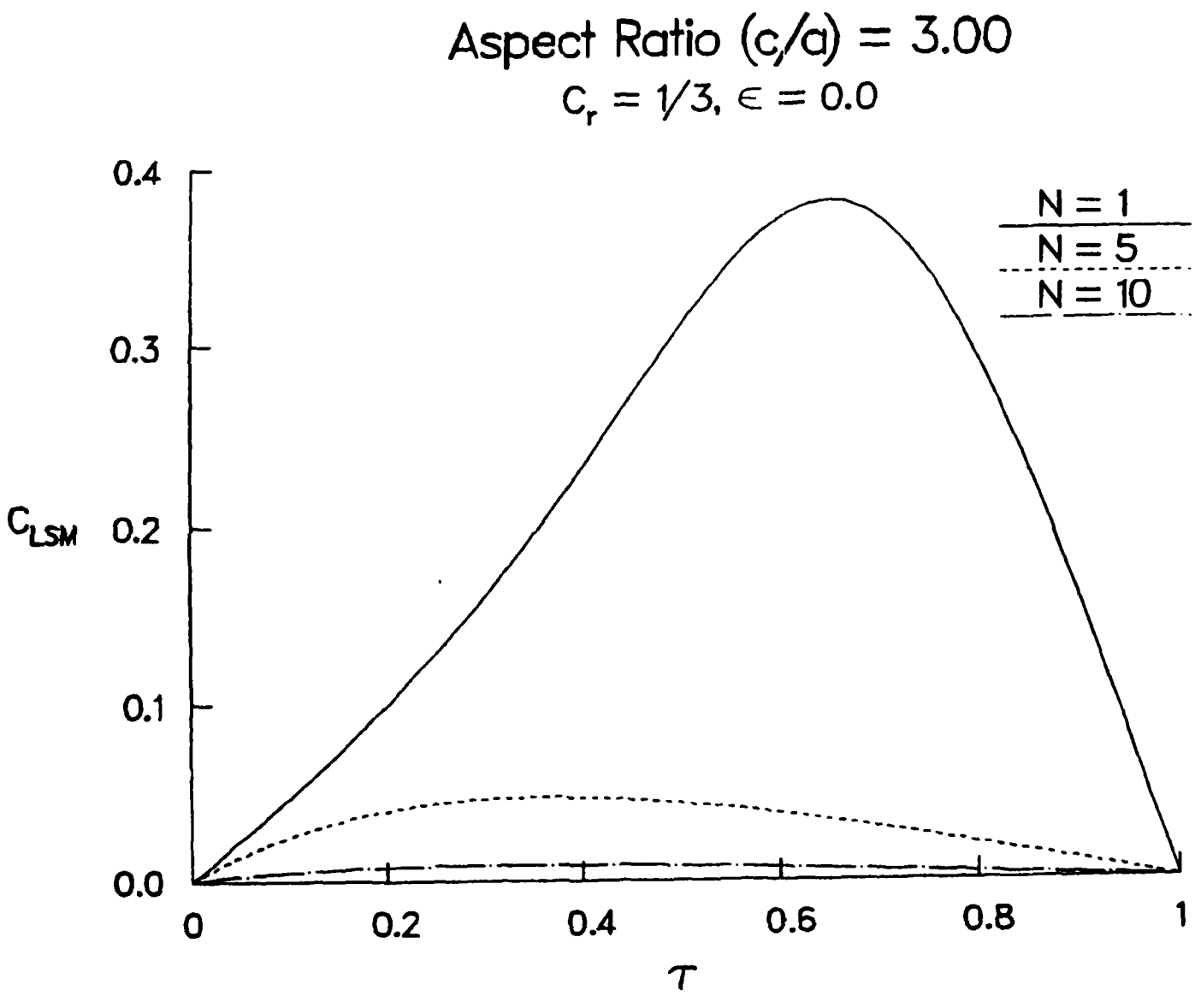

Figure 6. Comparison of C_{LSM} versus τ for f = 3, C_r = 3, ϵ = 0, N = 1, 5, 10 and C_{LSM} frozen.

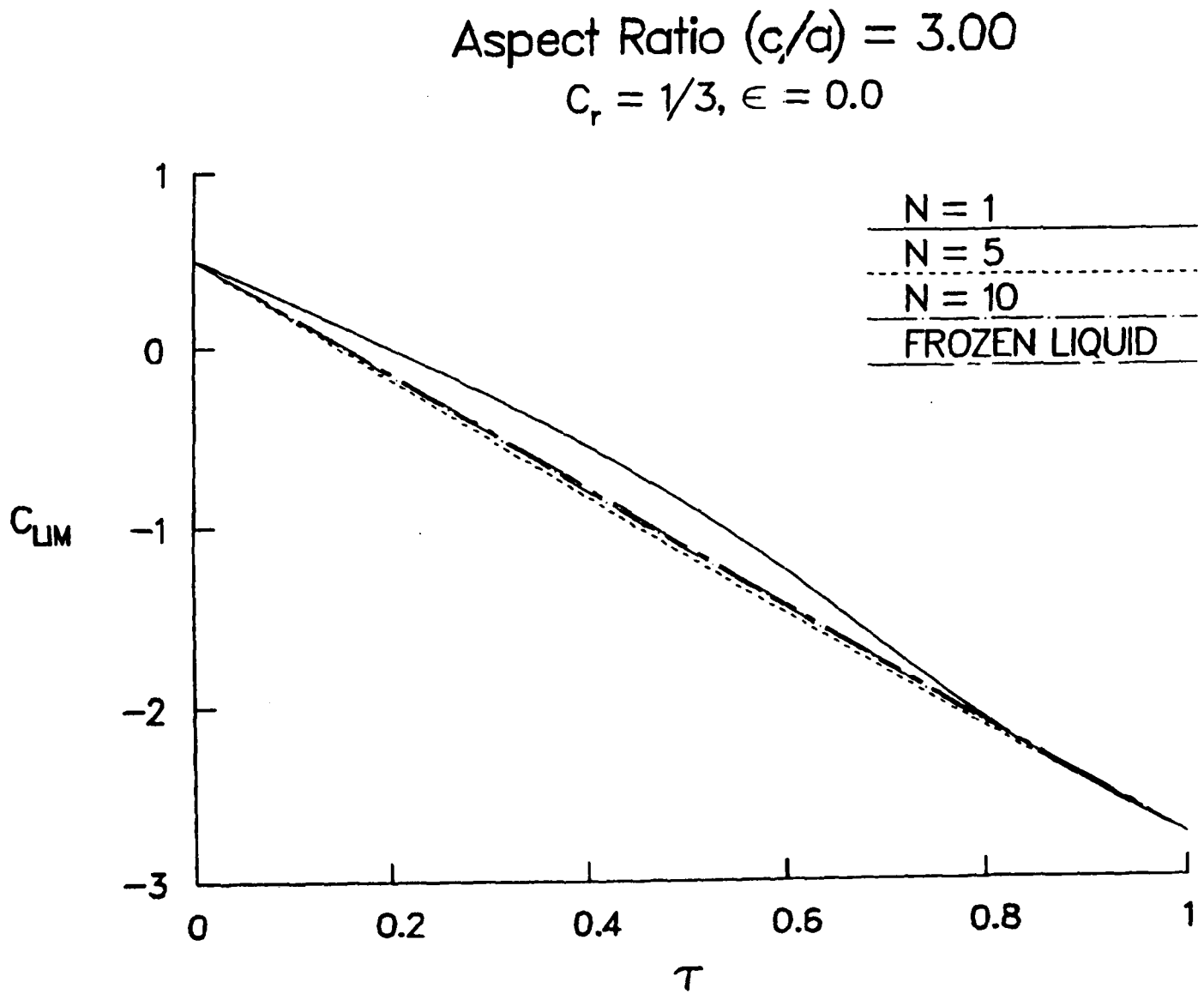

Figure 7. Comparison of C_{LIM} versus τ for f = 3, C_r = 3, ϵ = 0, N = 1, 5, 10 and C_{LIM} frozen.

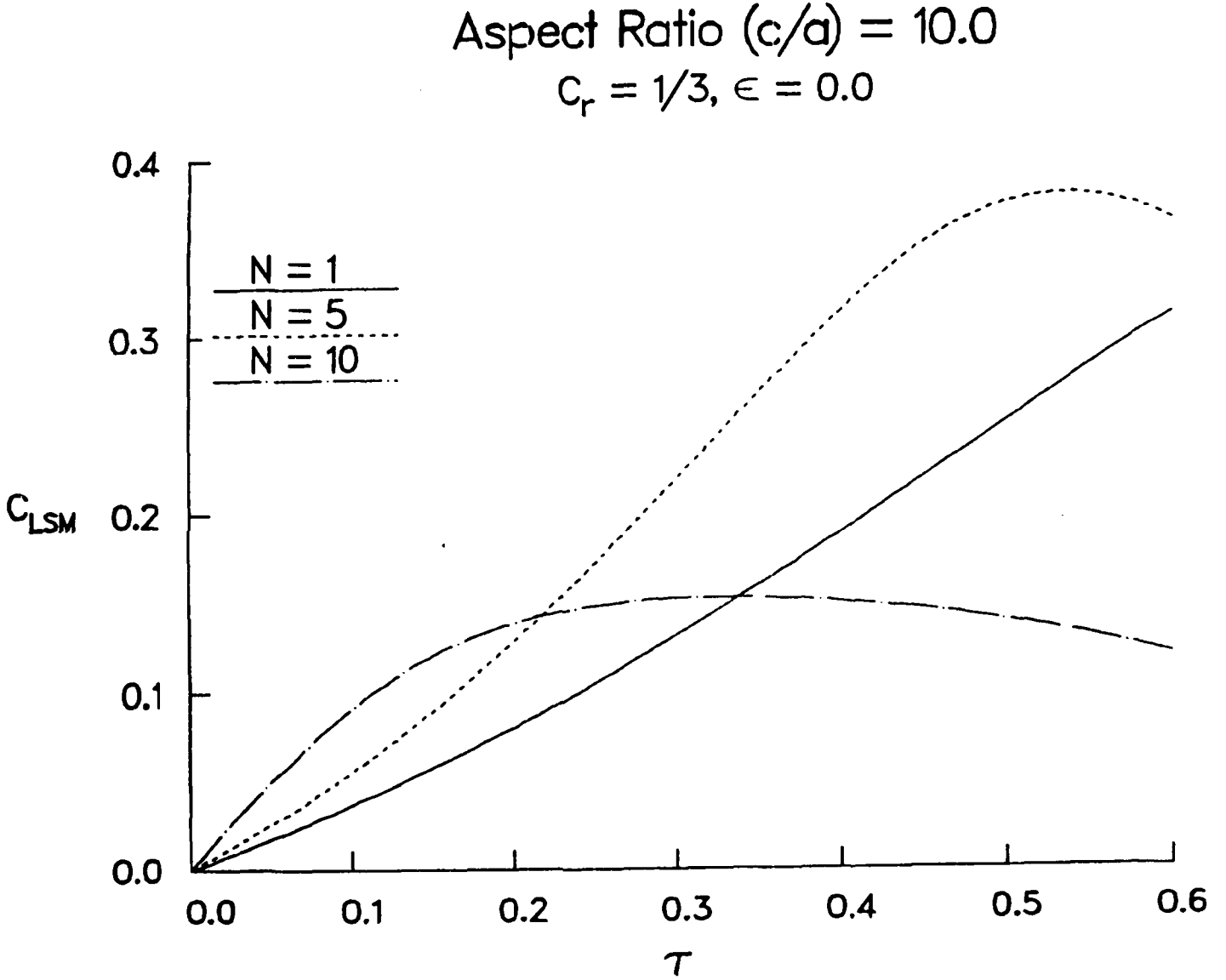

Figure 8. Comparison of C_{LSM} versus τ for f = 10, C_r = 3, ϵ = 0.0, N = 1, 5, 10.

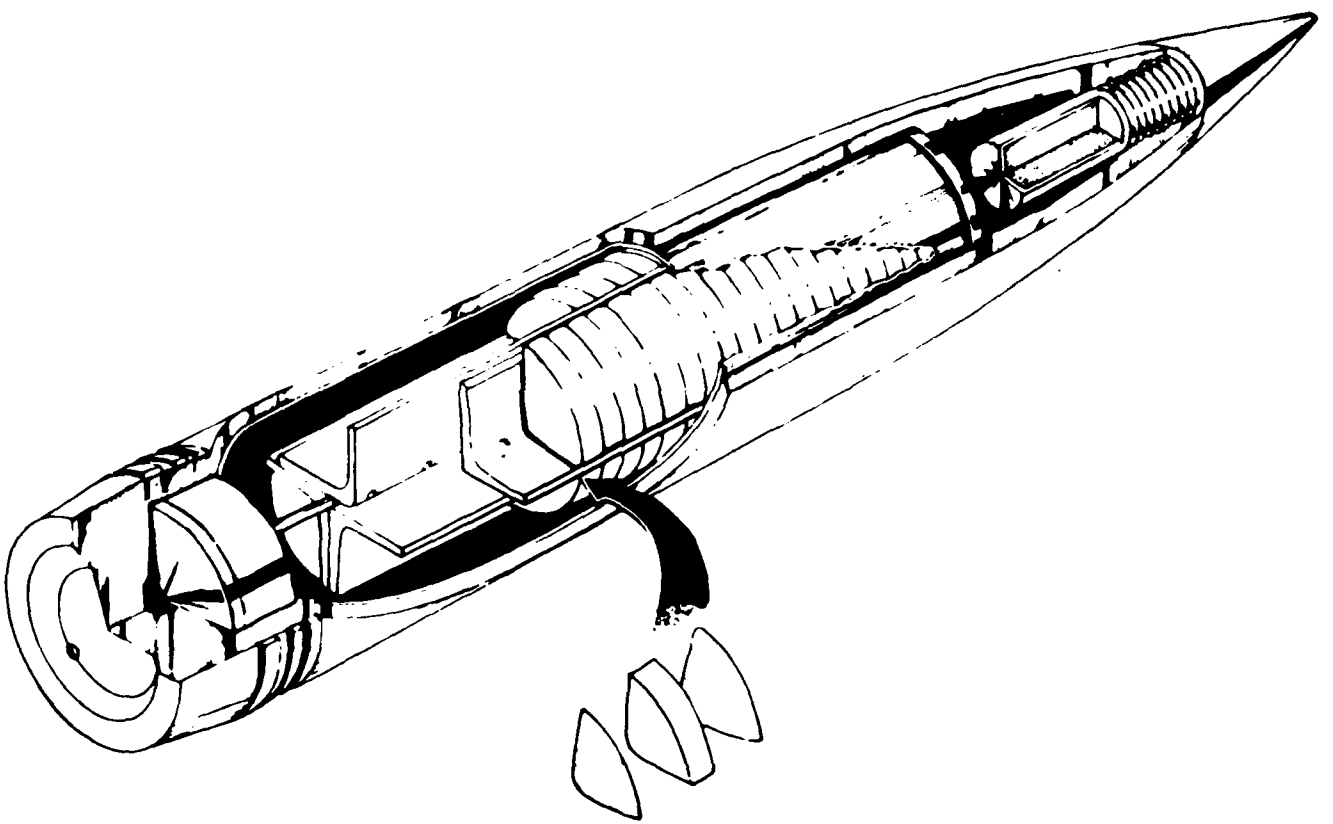

Figure 9. M825 improved smoke projectile.

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LIST OF SYMBOLS

a	Maximum radial distance of the liquid-filled container (for a cylinder, the radius, for a spheroid, the radial semi-axis)
C_{LIM_j}	Imaginary part of C_{LM_j}; coefficient representing the liquid moment that causes rotation in the plane of exp $i\phi_j$), j = 1,2
C_{LSM_j}	Real part of C_{LM_j}; coefficient representing the liquid moment that causes rotation out of the plane of exp ($i\phi_j$), j = 1,2
C_r	Drag coefficient
D_r	Drag term due porous media
$\vec{e}_x, \vec{e}_r, \vec{e}_\theta$	Unit vectors along the earth-fixed cylindrical axes
$\vec{e}_x, \vec{e}_y, \vec{e}_z$	Unit vectors along the earth-fixed Cartesian axes
f	Fineness ratio, c/a, for a cylinder
h	Distance of center of gravity along symmetry axis from geometric center
k	Axial wave number
$\hat{K}$	$K_{jo}e^{i\phi_{jo}}$ (j = 1,2)
K_j	$K_{jo}e^{\varepsilon_j \tau_j \phi}$ (j = 1,2)
K_{jo}	Value of K_j at t = 0
m_L	Mass of the liquid in the container $2\pi a^2 c\rho_L$ for a cylinder; $4\pi a^2 c\rho_L/3$ for a spheroid)
$M_{L\tilde{Y}} + M_{L\tilde{Z}}$	Transverse liquid moment in the aeroballistic nonrolling system
N	Number of products in the X factor, Eq. (3.19)
n_{XE}, n_{YE}, n_{ZE}	Earth-fixed components of a unit vector along the x-axis
p	Non-dimensional pressure perturbation as a function of r and x
r	Radial coordinate in the earth-fixed cylindrical (x,r,θ) system
$R\{\}$	Real part of {}

s	$(\varepsilon_j + i)\tau_j$ $\quad j = 1$ or 2
u,v,w	Non-dimensional velocity perturbation components in the earth-fixed cylindrical (x,r,θ) system
$\vec{V}$	Non-dimensional liquid velocity perturbation, Eq. (A.3)
V_x,V_r,V_θ	Components of $\vec{V}$ in the earth-fixed cylindrical (x,r,θ) system
x	Axial coordinate in the earth-fixed cylindrical (x,r,θ) system
XYZ	Missile-fixed axes, the X-axis along the projectile's axis of symmetry
$X\tilde{Y}\tilde{Z}$	Aeroballistic non-rolling axes, the $\tilde{Z}$-axis initially downward
$X_eY_eZ_e$	Earth-fixed axes, the X_e-axis along the velocity vector, Z_e downward
ε_j	$(\dot{K}_j/K_j)/\dot{\phi}_j$, non-dimensionalized damping; $j = 1,2$
θ	Azimuthal coordinate in the earth-fixed cylindrical (x,r,θ) system
ν	Kinematic viscosity of the liquid
$\tilde{\xi}$	$\hat{K}e^{s\theta}$
λ_k	Defined in Eq. (3.27)
ρ_L	Liquid density
τ_j	$\dot{\phi}_j/\dot{\phi}$, non-dimensionalized frequency; $j = 1,2$
$\dot{\phi}$	Spin rate with respect to inertial axis, assumed positive

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